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Abstract

The radiation cross section for relativistic suprathermal proton bremsstrahlung has been calculated using the Weizsäcker - Williams method. The method which is the natural one to use for this process, is briefly discussed and its relationship to the history of cosmic-ray research is mentioned. Curves representing the radiation spectra for protons of various energies are shown and an approximate expression for the spectrum, valid for photon energies greater than a few MeV, is given. This approximate spectrum is of the form such that the general analysis, presented by the author to the Calgary Cosmic - Ray Conference, may be applied. This analysis shows that an inverse power law spectrum of protons will produce, via the suprathermal proton bremsstrahlung process, an inverse power law spectrum of gamma rays that is one power <u>steeper</u> than that of the protons. This makes it quite unlikely, contrary to a previously published report, that this process can contribute significantly to cosmic gamma rays.

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Frank C. Jones

Suprathermal proton bremsstrahlung (SPB) has recently been considered as a possible source of cosmic x and γ rays. (Boldt and Serlemitsos 1969; Hayakawa 1969; Brown 1970 a, b). This process is just ordinary bremsstrahlung viewed in a frame where the electron is at rest and the heavy particle (proton) is moving.

The usual method of calculating the radiation cross section for SPB has been to view the process in the proton rest frame, treat it there as ordinary bremsstrahlung, and then transform back to the electron rest frame (lab frame). In the case of non-relativistic protons ($\beta \ll 1$) this approach is rather simple (for an explanation see Boldt and Serlemitsos 1969) and leads to the non-relativistic form of the Bethe-Heitler formula (Heitler 1954)

$$\frac{d\sigma}{dh\gamma} = \frac{\sigma_o}{\pi 137 h\nu} \left(\frac{mc^2}{T_o}\right) \ln \frac{[T_o^{\frac{1}{2}} + (T_o - h\nu)^{\frac{1}{2}}]^2}{h\nu} \tag{1}$$

where $\sigma_o = 665 \times 10^{-27}$ cm^2,

the Thomson cross section and T_o is the kinetic energy of the electron in the proton rest system

i. e. $T_o = \left(\frac{m}{M_p}\right) T_p$.

In the relativistic case ($\beta \approx 1$) the Doppler shift of the photon frequency is important and one (Brown 1970b) must use a bremsstrahlung cross section that includes the photon emission angle (Gluckstern and Hull 1953) and perform the appropriate Lorentz transformation to the electron rest frame. In a previous paper (Jones 1971) we showed that by far the

simplest and most natural method for calculating the SPB radiation cross section is the Weizsäcker - Williams method. (Weizsäcker 1934; Williams 1935; Heitler 1954; Jackson 1962). In this approach the process is viewed in the electrons rest frame; in this frame the electromagnetic field of the moving proton strongly resembles a plane pulse of radiation. This similarity may be exploited by considering the radiation process to be nothing more than a scattering of these (virtual) photons out of the pulse by the electron via the Compton scattering process.

In the original application (Weizsäcker 1934; Williams 1935) of this method one kept close track of the angles and energies of the scattered photons and then transformed back to the proton rest frame to obtain a spectrum that was quite consistant with the Bethe-Heitler formulas for bremsstrahlung. In our case, however, the electron rest frame <u>is</u> the lab frame and there is no need to keep track of scattering angles or to perform any Lorentz transformations at all. I think that this shows that the Weizsäcker - Williams method is the natural one to use in treating relativistic SPB. The results obtained by the author (Jones 1971) using this method differ considerably from those obtained previously using the conventional method (Brown 1970 b). It is my opinion that this reflects the ease and simplicity of the present method as compared to the complexity of the calculations required by the usual method. I would recommend reading Jackson (1962) for a complete discussion of the philosophy and methods of applying the Weizsäcker - Williams approach to a variety of problems.

Before discussing the results of the present calculations it should be pointed out that the Weizsäcker - Williams is intimately connected with a chapter of cosmic - ray history. When Bethe and Heitler (1934) first

published their theory of the interaction of electrons and photons with matter it was believed that the theory must break down when the electron energy got as high as a few GeV because cosmic ray data seemed to indicate that some energetic electrons did not radiate nearly as much as predicted by the Bethe - Heitler theory. On the other hand the work of Williams and Weizsäcker seemed to bolster the validity of the Bethe - Heitler formulas. Williams (1935) speculated that his approach might break down when the width of the Lorentz contracted proton field was of the order of the classical electron radius e^2/mc^2 or an electron energy of about 70 MeV. However, earlier Oppenheimer (1934) had dismissed Weizsäckers work in a rather condescending manner because it seemed to justify a theory that was controverted by observation.

As we all know now, of course, those "penetrating" electrons turned out to be μ mesons and the later development of the theory of electromagnetic cascades, incidentally by Oppenheimer (Carlson and Oppenheimer 1936) among others, proved to be a brilliant success for the theory of Bethe and Heitler and therefore also for that of Weizsäcker and Williams. The book Cosmic Rays by Rossi (1964) gives an interesting account of this development in cosmic-ray history.

The number of virtual photons (averaged over impact parameters) in the frequency range α_o to $\alpha_o + d\alpha_o$ in the electromagnetic field of the moving proton is given by (Jackson 1962)

$$N(\alpha_o)d\alpha_o = \frac{2}{\pi}\left(\frac{e^2}{\hbar c}\right)\frac{1}{\beta^2}\,\ln(A/\alpha_o)\,\frac{d\alpha_o}{\alpha_o} \tag{2}$$

where $\alpha_o = \hbar\omega_o/m_e c^2$

and $A = 1.123\,\gamma\beta^2 \exp(-\beta^2/2)$.

The spectrum is non-zero only for $\alpha_o < A$; actually for $\alpha_o \approx A$ equation (1) is not strictly correct and the spectrum goes to zero exponentially as $\exp(-2\alpha_o/\gamma\beta^2)$. This part of the spectrum does not contribute to the scattered spectrum in a significant amount since the Klein - Nishina cross section drops off at high energies so we shall ignore this complication.

The Klein - Nishina cross section for scattering a photon of energy α_o into an energy between α and $\alpha+d\alpha$ is given by

$$\frac{d\sigma(\alpha,\alpha_o)}{d\alpha} = \frac{\pi r_o^2}{\alpha_o^2}\left[\frac{1}{\alpha_o^2} + \frac{(2+\alpha-2/\alpha)}{\alpha_o} + \frac{(1-2\alpha)}{\alpha^2} + \frac{\alpha_o}{\alpha}\right] \tag{3}$$

where r_o is the classical electron radius $e^2/m_e c^2$ and α has the constraint $\alpha_o \geq \alpha \geq \alpha_o/(1+2\alpha_o)$.

The SPB cross section is now obtained by performing one simple integration

$$\frac{d\sigma(\alpha)}{d\alpha} = \int_{\alpha}^{\alpha_u} \frac{d\sigma(\alpha,\alpha_o)}{d\alpha} N(\alpha_o)\, d\alpha_o \tag{4}$$

where $\alpha_u = \mathrm{Min}\,(A, \alpha/(1-2\alpha))$. The above integral may be expressed in closed form in a straightforward manner. The final expression

is algebraically complicated and will not concern us here; it may be found in the previous paper of the author (Jones 1971).

Figure 1 shows plots of $d\sigma(\alpha)/d\alpha\ (2r_o^2/137)^{-1}$ for values of $\gamma = E_p/M_pc^2$ ranging from 1.1 to 40. The important fact to be noted about the SPB spectrum is that if α is somewhat larger than 1 ($\hbar\omega \gtrsim$ 5 Mev or so) the expression for $d\sigma(\alpha)d\alpha$ may be simplified considerably to

$$\left(\frac{d\sigma(\alpha)}{d\alpha}\right) \approx \left(\frac{2r_o^2}{137}\right)\alpha^{-2}\left[\frac{4}{3}\ \ln\ (A/\alpha) + (\frac{\alpha}{A} - 1) + \frac{1}{9}\ (\frac{\alpha^3}{A^3} - 1)\right] \tag{5}$$

If we define $X = \alpha/A$ we have $\left(\frac{d\sigma(\alpha)}{d\alpha}\right) d\alpha \approx F(X)\ d\alpha/A^2$

$$F(X) = \left(\frac{2r_o^2}{137}\right)X^{-2}\left[(X-1) + (X^3-1)/9 - \frac{4}{3}\ \ln X\right]. \tag{6}$$

In a paper presented at the cosmic - ray conference in Calgary (Jones 1968) the present author showed that if a radiation spectrum produced by a particle of energy $\gamma = E/Mc^2$ could be expressed as $N(\alpha)d\alpha = F(\alpha/\alpha_c)\ d\alpha/(\alpha_c)^n$ where the critical photon energy α_c is related to the primary particle energy by $\alpha_c = k\gamma^p$ then the secondary spectrum arising from an inverse power law primary spectrum $j(\gamma) = k'\gamma^{-m}$ can be shown to have a particularly

simple form

$$d\alpha \int N(\alpha,\gamma)\ j\ (\gamma)\ d\gamma \approx K\ \alpha^{-\Gamma} d\alpha$$

where

$$\Gamma = n + (m-1)/p.$$

In the present case $\alpha_c = A \propto \gamma$ so p=1 and n=2. This gives $\Gamma = m+1$ so the SPB photon spectrum of gamma rays of energy greater than about 5 MeV arising from an inverse power law spectrum of protons is always one power steeper than the proton spectrum.

Since we would expect any spectrum of 20 GeV or greater potons to be at least as steep as γ^{-2} the resulting γ-ray spectrum would be at least as steep as α^{-3}. Thus it appears unlikely that SPB makes a significant contribution to cosmic gamma rays above a few MeV.

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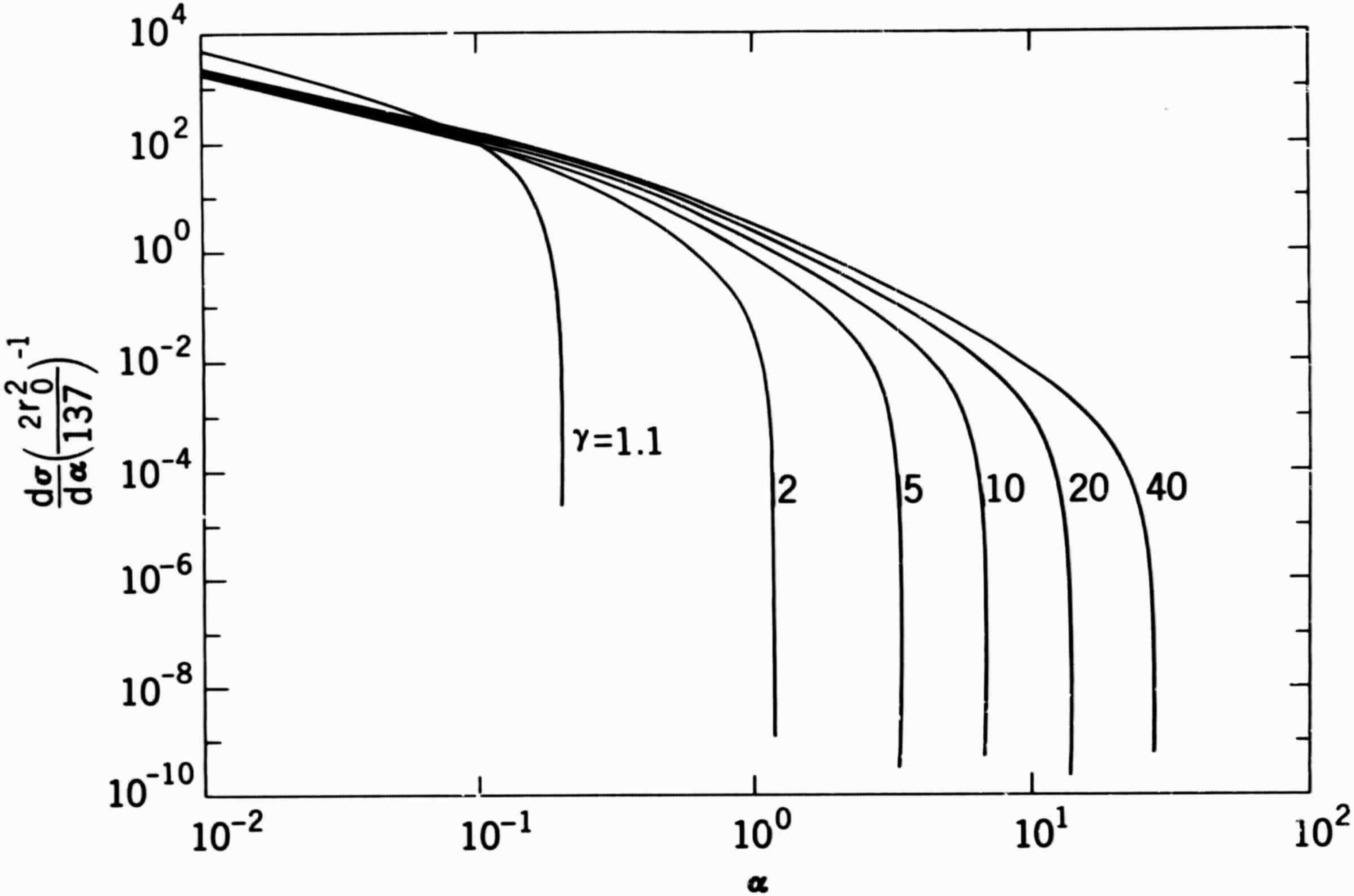
Figure Captions

Figure 1 - Plot of the suprathermal proton bremsstrahlung (SPB) cross section as a function of $\alpha = \hbar\omega/m_e c^2$ for various values of $\gamma = Ep/MpC^2$. The cross section is plotted in units of $2ro^2/137$ (= 1.16×10^{-27} cm^2) per unit α.



$\frac{d\sigma}{d\alpha}\left(\frac{2r_0^2}{137}\right)^{-1}$
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$\gamma=1.1$
2
5
10
20
40
10^{-2}
10^{-1}
10^0
10^1
10^2
α